Title: **EE–I–EE–I–O! An Exploration in Linear and Quadratic Functions**

Brief Overview:

This unit is designed to give students an opportunity to apply real world skills to maximize area when given a fixed perimeter. Students will need an understanding of plotting points on a coordinate grid, area and perimeter formulas, function notation and use substitution to solve for unknowns.

In the first lesson, students will be exploring to find the dimensions which will yield the maximum area given a fixed perimeter. In the second lesson, students will develop the linear formula which creates the length/width table and examine and interpret the parabola created by the area data. Teachers can decide based on needs of their students if they would like to develop the quadratic equation by hand or using the graphing calculator (or Excel). In the third lesson, students will extend their understanding to a new situation where they must also maximize area given a perimeter, but new constraints are added.

NCTM Content Standard/National Science Education Standard:

- The student will represent patterns and/or functional relationships in a table, as a graph, and/or by mathematical expression.
- The student will describe the graph of a non–linear function and discuss its appearance in terms of the basic concepts of maxima and minima, zeros (roots), rate of change, domain and range, and continuity.
- The student will determine the equation for a line, solve linear equations, and/or describe the solutions using numbers, symbols, and/or graphs.

Grade/Level:

7 – 9 Grade, Algebra 1

Duration/Length:

Three to four 45 minute sessions

Student Outcomes:

Students will:

- Calculate area given a fixed perimeter.
- Create a table of values using the formula for perimeter and area.
- Write a linear relationship based on information from a table.
- Use substitution to evaluate systems of equations.
- Graph linear and quadratic functions.
- Identify and interpret zeroes and maximums of a function.

Materials and Resources:

- Per Student:
- Graph paper (1 cm grid)
- Dot Paper (square)
- Graphing Calculator
- String/yarn
- Toothpicks
- Cylinder with string attached
- Worksheets
    - Area and Perimeter Warm-Up
    - Quadrilateral Quandary
    - Mrs. McDonald's Garden Part I
    - EI-EI-O Warm-Up
    - Mrs. McDonald's Garden Part 2
    - Mrs. McDonald's Garden Part 3
    - Cakes and Quadratics Exit Ticket
    - Patterns Warm-Up
    - Woof! Woof! Launch
    - EI-EI-O Activities
        - Cow Pasture
        - Pig Pen
        - Billy the Goat

Development/Procedures:

Lesson 1

Pre–assessment – Prior to this lesson, students have already used area and perimeter formulas. To assess their knowledge of these skills have students complete the "Area and Perimeter Review" warm–up.

Launch –Distribute the "Quadrilateral Quandry", which has the students make observations about two different rectangles. Prompt the students to consider comparing the area and perimeter of the rectangles. Students should identify the fact that area is independent from perimeter. Challenge the students to construct a rectangle with the same perimeter with a different area.

Teacher Facilitation – Give students the Student Resource Packet and present the problem "Mrs. McDonald's Garden". Provide the necessary manipulatives: string, toothpicks, and/or graph paper. Explain to students that they will eventually share their results with the class and that today's assessment will be performance based. This activity is open–ended to allow for various strategies.

Student Application – Allow students 3 – 5 minutes to brainstorm ways to solve the problem individually. Have students form groups of 3 – 4 and discuss their problem solving strategies. Once students have decided which strategy they prefer allow sufficient time to implement and organize their chosen strategy to identify the dimensions to maximize area.

Embedded Assessment – As students are cooperatively working circulate through groups to monitor progress and trouble shoot as needed. Once students have found their dimensions have them explain their strategy to the class. Students will be assessed on both their ability to work collectively and use of problem solving skills.

Reteaching/Extension –

- Presentations of strategies will serve as a reteaching. Encourage students to ask questions to clarify any misunderstandings.
- Extend the lesson by having the students to research how much space different animals need.

Lesson 2

Pre–assessment– Assess the students' ability to complete linear and nonlinear patterns, solving literals, and evaluating expressions using the "EI-EI-O" warm-up. Note that the.students will also need an understanding of solving systems of equations using substitution for this lesson, which is not included in the preassessment.

Launch– Review the problem presented in the previous lesson. Explain to students that today will focus on the table of values created yesterday. Note that if no groups created a table, provide a blank table for the students to complete.

Teacher Facilitation – Re–form the same groups from the previous lesson. Distribute "Mrs. McDonald's Garden Day Part 2" and allow students approximately fifteen minutes to complete the worksheet. As a class, discuss their findings, focusing on how the interpretation of the graph, slope and *y* – intercept.

Distribute "Mrs. McDonald's Garden Part 3" and allow fifteen minutes to complete the worksheet. As a class, discuss their findings, focusing on the non–linear (quadratic) relationship in the graph. Focus instruction on the interpretation of the graph, and how the students determined the formula for finding area. Identify this as a quadratic equation. Define the terms maximum and relative maxima. Ask the students if a graph can have a minimum. Connect identifying the maximum on the graph as the solution to their original problem from yesterday, helping them to make the connection between the graph and the real world scenario. Instruct the students to add the new vocabulary and definitions (maximum, minimum, zeroes, quadratic function, parabola) in their notes.

Using graphing calculators, work as a class to create the scatter plot of width versus area. Compare the graph found on the calculator to the one the students' generated by hand (length to area). Have the students identify the zeroes and maximum using the tracing and calculate features of the calculator. Discuss why the graphs are so similar. Talk about why this graph contains negative values and if these make sense in the context of this problem. Review function notation with students and discuss that for this problem the area is a function of length/width.

Student Application – Students will work cooperatively to complete the handout. They will also be able to contribute to class discussion in the development of the quadratic equation.

Embedded Assessment – Distribute "Cakes and Quadratics" exit ticket to assess student understanding of the difference between a linear and a quadratic function.

Reteaching/Extension

- Challenge the students to notice that the pattern of the pattern is linear.

- As time permits work with students to create the equation for the width based on the perimeter equation then leading them to finding the new equation for area based only on the length. Discuss the linear regression feature and the quadratic regression feature of the graphing calculator.
- Teachers can return to this lesson when multiplying and factoring polynomials: $A = l(18 - l) = 18l - l^2$.

Lesson 3 Pre–assessment – Assess the students' recall of the properties of quadratic functions, and the difference between a linear and a quadratic function with the "Patterns" warm-up.

Launch – Present the students with a new situation involving area of a circle. Allow students 5–8 minutes to complete "Woof! Woof!", working with a partner as appropriate. Invite students to share their solutions, and ask how this relates to Mrs. McDonald's Garden.

Teacher Facilitation – Set up the classroom for differentiated instruction. Group the students homogenously and give students problems from the "EI-EI-O" activities appropriate to each level or group students heterogeneously to allow for learning from each other during the "EI-EI-O" activities. Present these activities by first explaining to the students that Old McDonald was inspired by Mrs. McDonalds amazing new garden and he decided to redo all the fencing on the farm. Give each group a different problem; allow them to use the same manipulatives as day 1, and/or graphing calculators to solve their problem as part of "Old McDonald's Farm". Explain to students that each group has a different problem and they will eventually share their results with the class. There are three different problems, and depending upon individual class sizes, two groups may need to receive the same problem. These groups could check their results together before presenting to the class.

Student Application – Student will work in groups to solve a new real world problem. They will need to apply the skills and strategies they learned over the past two days to complete the assignment.

Embedded Assessment – As students are cooperatively working circulate through groups to monitor progress and trouble shoot as needed. Once students have found their dimensions have them explain their strategy to the class. Students will be assessed on both their ability to work collectively and use of problem solving skills.

Reteaching/Extension –

- Presentations of strategies will serve as a reteaching. Encourage students to ask questions to clarify any misunderstandings.
- Have students find the total area needed for Old McDonald's farm.
- Follow up questions can include finding the total cost of fencing and the smallest possible area of Old McDonald's farm after all groups have presented.

Summative Assessment:

The teacher should informally assess the students as they work cooperatively on all activities and participation in group discussions. Students will be assessed on their ability to complete the group project on day 3. They should use an appropriate problem solving strategy to maximize the area requited by each animal and incorporate the skills introduced in this unit.

A formal assessment is not included in this unit, however, items from the Maryland High School Assessment (HSA) for Algebra and Data Analysis have been included from the 2004-2008 tests. These items can be used at the teacher's discretion.

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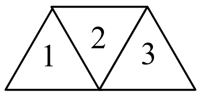
Area and Perimeter Review
Warm-Up

Name: ______________________
Date: ______________________

1. Sam used straws to form a pattern of triangles shown below. If he continues this pattern, what is the total number of straws he needs to form 6 triangles?



A. 9 **B.** 13

C. 14 **D.** 18

2. The table below shows the price of a pizza with toppings.

No. of Toppings, *x*	0	1	2	3
Price of Pizza, *y*	\$6.00	\$7.25	\$8.50	\$9.75

Which equation below represents the relationship?

A. $y = 1.25x - 6$ **B.** $y = 1.25x + 9.75$ **C.** $y = 1.5x + 6$ **D.** $y = 1.25x + 6$

3. The radius of a pizza is 9 inches, calculate the area where $A = \pi r^2$. (Let $\pi = 3.14$)

A 81 in^2 **B** 254.34 in^2 **C** 28.26 in^2 **D** 280 in^2

4. An architect uses blocks to design stairs for buildings. The number of blocks increases as the number of steps increases. Complete the table below.

Level Number	1	2	3	4	5	6	7
Number of Blocks	1	3	6	10			

How many blocks will be needed for a model of Level 10?

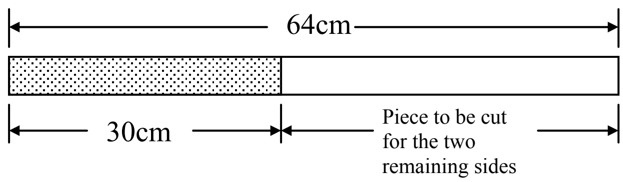
5. To make a model of the Guadeloupe River bed, Hermie used 1 inch of clay for 5 miles of the river's actual length. His model river was 50 inches long. How long is the Guadeloupe River?

A. 5 miles **B.** 10 miles **C.** 100 miles **D.** 250 miles

EI-EI-O
Warm-Up

Name: ________________________
Date: ________________________

1. A piece of lumber that measured 64cm in length was cut into three pieces to build the sides of a box shaped like an isosceles triangle. The longest piece, 30cm, was cut first. If the entire piece of lumber is to be used, what was the length of the other two pieces?



2. The length of a soccer field is 20 greater than 3 times the width. The perimeter of the field is 440 feet.

- Write an expression for the length of the field in terms of the width, w.

- Find the dimensions of the soccer field.

3. The area of a triangle can be found using the formula

$$A = \frac{bh}{2}, \text{ where} \left\{ \begin{array}{l} b \text{ is the base} \\ h \text{ is the height} \end{array} \right\}$$

Solve the formula for h.

4. Which table represents a linear pattern?

A.

x	0	1	2	3
y	3	4	6	9

B.

x	0	1	2	3
y	3	9	27	81

C.

x	0	1	2	3
y	3	4	7	12

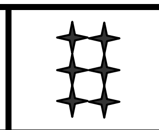
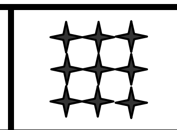
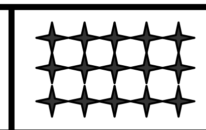
D.

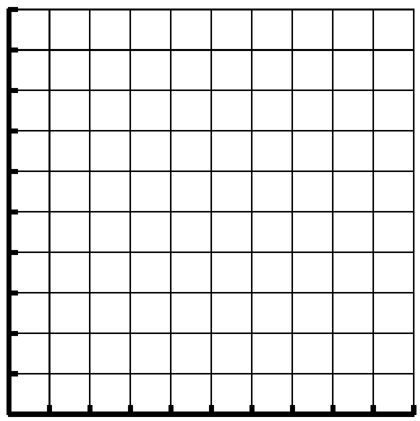
x	0	1	2	3
y	3	7	11	15

Patterns Name: ________________________
Warm-Up Date: _________________________

1. Look at the pattern to the right.

Complete the table and graph the data.

			
Stage	1	2	3

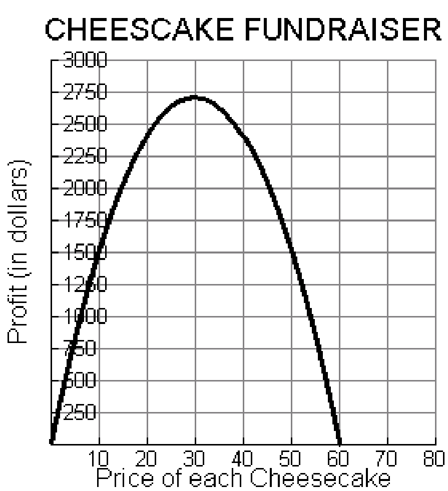


Stage	1	2	3	4	5
No. of Figures	6				

Is the graph linear? Why or why not?

Identify the numerical pattern in the data.

2. In order to raise funds, the high school cheerleading squad is selling gourmet cheesecakes. The profit P can be modeled by the function, $P(x) = -3x^2 + 180x + 8$, where x is the price of each gourmet cheesecake.



- What is the value of $P(10)$? What does this represent in the context of the problem?
- Estimate the maximum profit the cheerleading squad could make selling the cheesecakes. What is the price of the cheesecake to make the profit?
- What price would allow the cheerleading squad to only "break even"? Explain how you determined your answer.

Teacher's Key All Warm–Ups

Area and Perimeter Review Warm–Up 1

1. A
2. D
3. B
4.

Level Number	1	2	3	4	5	6	7	10
Number of Blocks	1	3	6	10	15	21	28	55

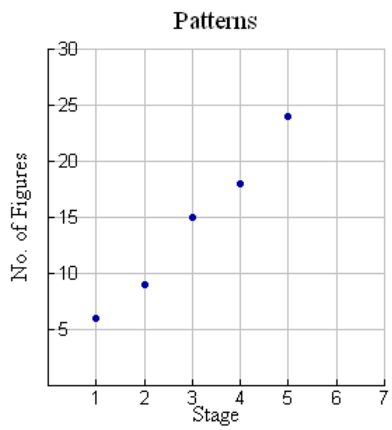
5. D

EI-EI-O Warm–Up

1. 17 cm
2. $l = 3w + 20$ $w = 50$ ft $l = 170$ ft
3. $h = \dfrac{2A}{B}$
4. D

Patterns Warm-Up

1.



Stage	1	2	3	4	5
No. of Figures	6	9	15	18	24

This is not linear because there is no constant rate of change.

The pattern alternates between adding three and adding six.

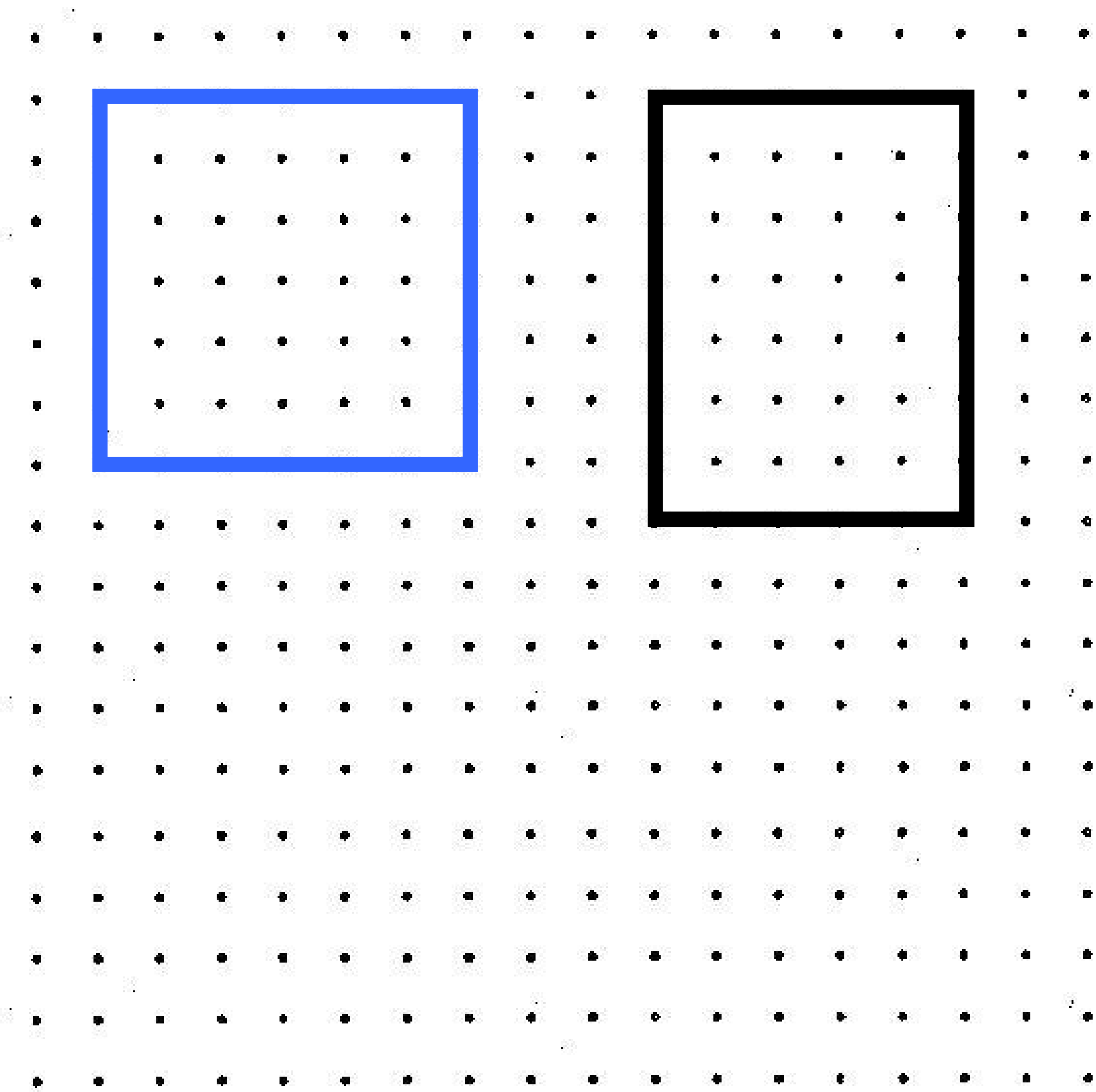
2.

- $P(10)$= 1,500 (1,508 if calculated using the formula) This means if the cheerleaders sell 10 cheesecakes they will have a profit of $1,500.
- The maximum profit is about $2750, the cheesecakes would cost $30.
- $60 would make the cheerleaders break even because this is a zero of the function.

Quadrilateral Quandary
Launch

Name: ______________________
Date: ______________________

Consider the two quadrilaterals below.



a. Compare the two quadrilaterals, and record your observations below.

____

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b. Draw another quadrilateral on the grid above which reflects your observations. Be prepared to share.

Mrs. McDonald's Garden
Part I

Name: ___________________
Date: ____________________

Mrs. McDonald is sick of having rabbits eat away at her best vegetables. She is building a new garden and has purchased 36 ft of fencing to keep out the rascally rabbits. She wants to maximize the area of her garden. Help her find the dimensions (length and width) to create her dream garden.

Step 1: Brainstorm ideas on how to solve the problem. Be prepared to share your ideas with your group.

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Step 2: Discuss your ideas with your group and decide on a plan of action. Use the space below to organize your data. You can use any manipulatives you feel are appropriate.

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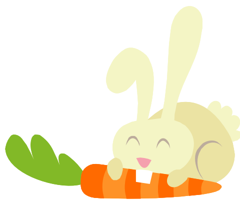
Mrs. McDonald's Garden
Part I

Name: _ANSWER KEY____
Date: ___________________



Mrs. McDonald is sick of having rabbits eat away at her best vegetables. She is building a new garden and has purchased 36 ft of fencing to keep out the rascally rabbits. She wants to maximize the area of her garden. Help her find the dimensions (length and width) to create her dream garden.

Step 1: Brainstorm ideas on how to solve the problem. Be prepared to share your ideas with your group.

Step 2: Discuss your ideas with your group and decide on a plan of action. Use the space below to organize your data. You can use any manipulatives you feel are appropriate.

Students could create a table, graph, use guess and check or any other mathematically sound strategy. Day 2 will focus on identifying the linear equation from the table as well as introducing the quadratic equation and the aspects of a parabola.

Length (ft)	**Width (ft)**	**Area $(ft)^2$**
0	18	**0**
1	17	**17**
2	16	**32**
3	15	**45**
4	14	**56**
5	13	**65**
6	12	**72**
7	11	**77**
8	10	**80**
9	9	**81**
10	8	**80**
11	7	**77**
12	6	**72**
13	5	**65**
14	4	**56**
15	3	**45**
16	2	**32**
17	1	**17**
18	0	**0**

Mrs. McDonald's Garden
Part 2

Name: ______________________________
Date: _______________________________

1. The table to the right lists all possible dimensions of a rectangular garden whose perimeter is 36 ft. Using what you know about functions, identify important relationships in the data.
   - Are there any evident patterns in the data?
   - Is there a consistent rate of change?
   - Is there a direct relationship between the length and the width?

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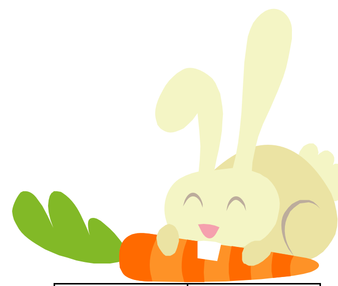

Length	Width
0	18
1	17
2	16
3	15
4	14
5	13
6	12
7	11
8	10
9	9
10	8
11	7
12	6
13	5
14	4
15	3
16	2
17	1
18	0

2. In the graph below, *sketch* what you think the graph of the data would look like. Use mathematics to explain your thinking. Use words, symbols, or both in your explanation.

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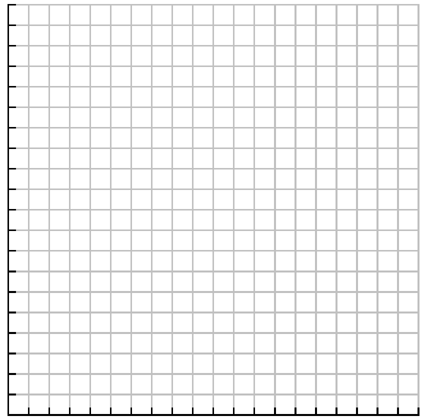
3. Use the grid below to make an accurate graph of the data relating the length to the width of the rectangular garden.



4. What type of function is represented by the graph? Use what you know about functions to justify your answer. Use words, symbols, or both in your justification.

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5. What is the slope of the function? ______________________________

6. What does the slope mean in the context of this situation?

____

7. What is the y – intercept of your function? ______________________________

8. What does the y – intercept mean as it relates to the problem?

____

9. Determine the equation of the linear function which fits this data.

____

10. Use your equation to find the width if the length of the garden is 20 ft. Explain why this answer is not realistic within the context of the problem.

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Mrs. McDonald's Garden
Part 2

Name: ___ANSWER KEY________
Date: __________________________

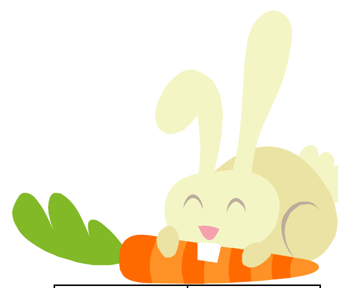
1. The table to the right lists all possible dimensions of a rectangular garden whose perimeter is 36 ft. Using what you know about functions, identify important relationships in the data.

- Are there any evident patterns in the data?
- Is there a consistent rate of change?
- Is there a direct relationship between the length and the width?

Answers may vary. Student answers may include:

- As the length increases, the width decreases.
- The width is decreasing at a constant rate of –1 as the length increases by 1.
- There is a negative correlation.

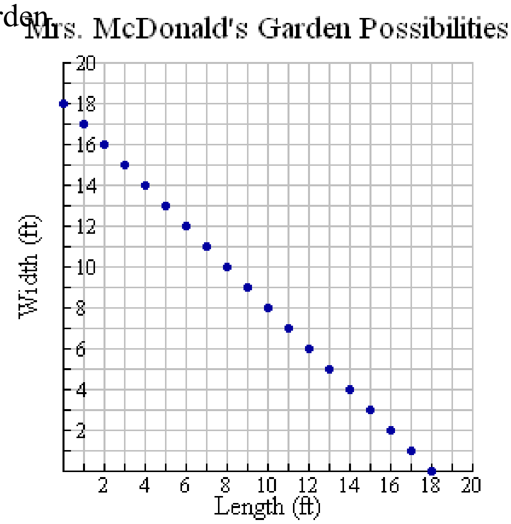
2. In the graph below, *sketch* what you think the graph of the data would look like. Use mathematics to explain your thinking. Use words, symbols, or both in your explanation.



Length	Width
0	18
1	17
2	16
3	15
4	14
5	13
6	12
7	11
8	10
9	9
10	8
11	7
12	6
13	5
14	4
15	3
16	2
17	1
18	0

Answers may vary, but graph should be linear.

3. Use the grid below to make an accurate graph of the data relating the length to the width of the rectangular garden.



4. What type of function is represented by the graph? Use what you know about functions to justify your answer. Use words, symbols, or both in your justification.

The graph represents a linear function. This is true because of the constant rate of change in the variables.

5. What is the slope of the function? ______ $m = -1$ ______

6. What does the slope mean in the context of this situation?
As the length increases by one foot, the width will decrease by a foot.

7. What is the y – intercept of your function?
____ $b = 18$ ______

8. What does the y – intercept mean as it relates to the problem?
If there was no length to the garden, the width would be 18 ft long.

9. Determine the equation of the linear function which fits this data.
___ $y = -x + 18$ ______

10. Use your equation to find the width if the length of the garden is 20 ft. Explain why this answer is not realistic within the context of the problem.

If the length is 20 feet, the width would be –2 ft. While the equation is giving a correct value, this value is not realistic because widths cannot be negative.

Mrs. McDonald's Garden
Part 3

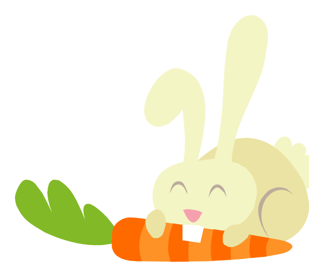
Name: ______________________
Date: ______________________

1. The table on the right compares possible lengths and widths of a rectangular garden with a perimeter of 36 ft. Find the corresponding areas for each of the dimensions in the chart.

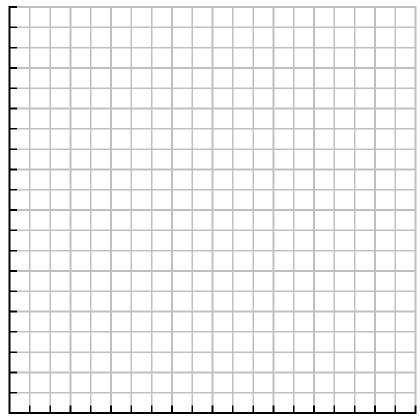
Length (ft)	Width (ft)	Area $(ft)^2$
0	18	
1	17	
2	16	
3	15	
4	14	
5	13	
6	12	
7	11	
8	10	
9	9	
10	8	
11	7	
12	6	
13	5	
14	4	
15	3	
16	2	
17	1	
18	0	

2. Using what you know about functions, identify important relationships in the data.
   - Are there any evident patterns in the data?
   - Is there a consistent rate of change?
   - Is there a direct relationship between the length and the width?

3. Mrs. McDonald would like to compare the length of the garden with the corresponding area. In the graph below, *sketch* what you think the graph of the data would look like. Use mathematics to explain your thinking. Use words, symbols, or both in your explanation.



4. Use the grid below to make an accurate graph of the data relating the length to the width of the rectangular garden.



5. What type of function is represented by the graph? Use what you know about functions to justify your answer. Use words, symbols, or both in your justification.

6. What is the coordinate of the maximum point?

7. What does the coordinate of the maximum point mean in the context of this situation?

8. What are the coordinates of the x – intercept (zeros)?

9. What do the coordinates of the x – intercept (zeros) mean in the context of this situation? Explain why these points are mathematically not logical in the context of the situation. Use words, symbols, or both in your explanation.

10. Determine an equation relating the area to the length and width of the garden.

Mrs. McDonald's Garden
Part 3

Name: ___ANSWER KEY________
Date: ____________________________

1. The table on the right compares possible lengths and widths of a rectangular garden with a perimeter of 36 ft. Find the corresponding areas for each of the dimensions in the chart.

Length (ft)	**Width (ft)**	**Area $(ft)^2$**
0	18	0
1	17	17
2	16	32
3	15	45
4	14	56
5	13	65
6	12	72
7	11	77
8	10	80
9	9	81
10	8	80
11	7	77
12	6	72
13	5	65
14	4	56
15	3	45
16	2	32
17	1	17
18	0	0

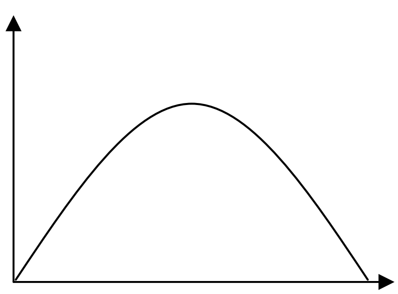
2. Using what you know about functions, identify important relationships in the data.

- Are there any evident patterns in the data?
- Is there a consistent rate of change?
- Is there a direct relationship between the length and the width?

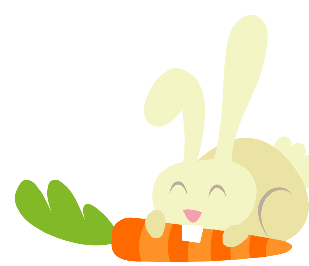
Answers may vary. Student answers may include:

- The area increases at a non constant rate to a maximum point then it decreases.
- The numbers repeat.

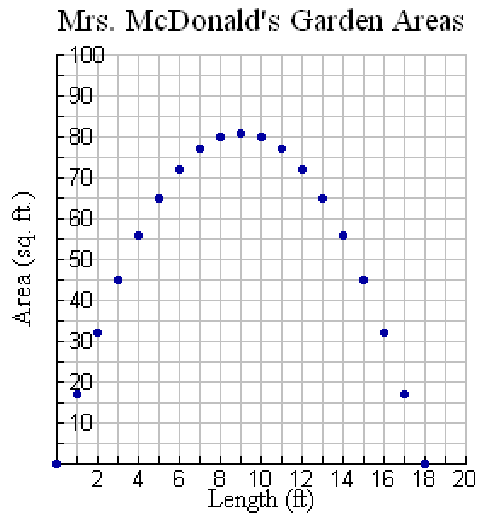
3. Mrs. McDonald would like to compare the length of the garden with the corresponding area. In the graph below, *sketch* what you think the graph of the data would look like. Use mathematics to explain your thinking. Use words, symbols, or both in your explanation.



Answers may vary, but graph should be quadratic.



4. Use the grid below to make an accurate graph of the data relating the length to the width of the rectangular garden.



5. What type of function is represented by the graph? Use what you know about functions to justify your answer. Use words, symbols, or both in your justification.

Answers may vary due to students' prior knowledge. Possible answers include:

- The graph is a parabola.
- It is a quadratic function.
- It is non-linear.

6. What is the coordinate of the maximum point?

$(9, 81)$

7. What does the coordinate of the maximum point mean in the context of this situation?

The rectangular garden will reach its maximum area of 81 sq. ft. with a length (and width) of 9 ft. This is a square.

8. What are the coordinates of the x – intercept (zeros)?

$(0, 0)$ and $(18, 0)$

9. What do the coordinates of the x – intercept (zeros) mean in the context of this situation? Explain why these points are mathematically not logical in the context of the situation. Use words, symbols, or both in your explanation.

When the length of the fence is zero or 18, your area will be zero. Lengths and widths cannot be zero, and neither can areas.

10. Determine an equation relating the area to the length and width of the garden.

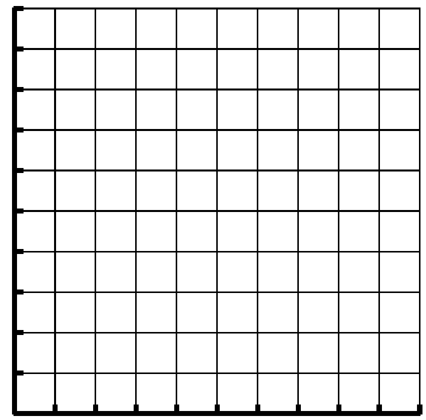
$y = x(18 - x)$, where x: length, $(18 - x)$: width, and y: area

Cakes and Quadratics
Exit Ticket

Name: ________________________
Date: _________________________

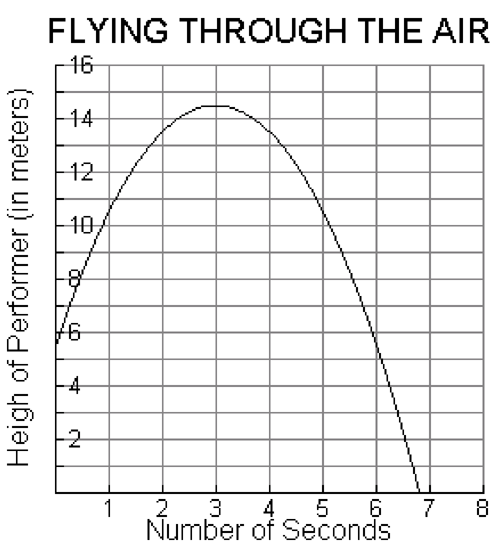
1. The table below represents the temperature of a cake as it cools.

Time(min)	1	2	3	4	5
Temp.(F°)	350°	346°	342°	338°	334°



- Is the relationship linear? Why or why not?
- Write an equation for the data.
- Graph the data.
- What will the temperature of the cake be after 15 minutes?

2. A circus performer is fired from a cannon. The function, $h(t) = 5.5 + 6t - t^2$, shows the height in meters, h, of the performer after t seconds.



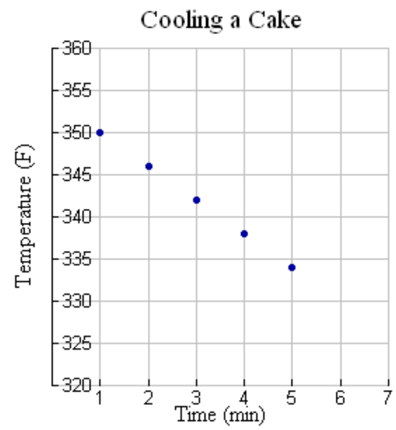
- What does the value of $h(4)$ represent in the context of the problem?
- Approximate the maximum height of the performer.
- About how long does it take for the performer to reach his/her maximum height?
- Approximately how long is the performer in the air before landing?

Cakes and Quadratics
Exit Ticket

Name: ___ANSWER KEY________
Date: ___________________________

1. The table below represents the temperature of a cake as it cools.

Time(min)	1	2	3	4	5
Temp.(F°)	350°	346°	342°	338°	334°



- Is the relationship linear? Why or why not?

  Yes, there is a constant rate of change of $\frac{-4°}{1\text{ min}}$

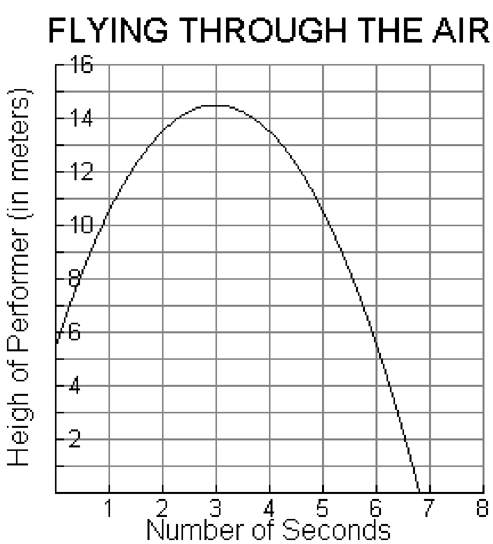
- Write an equation for the data.
  $y = 350 - 4x$

- Graph the data.

- What will the temperature of the cake be after 15 minutes?

  290 degrees

2. A circus performer is fired from a cannon. The function, $h(t) = 5.5 + 6t - t^2$, shows the height in meters, h, of the performer after t seconds.



- What does the value of $h(4)$ represent in the context of the problem?
  After 4 seconds, the performer is 13.5 meters in the air.

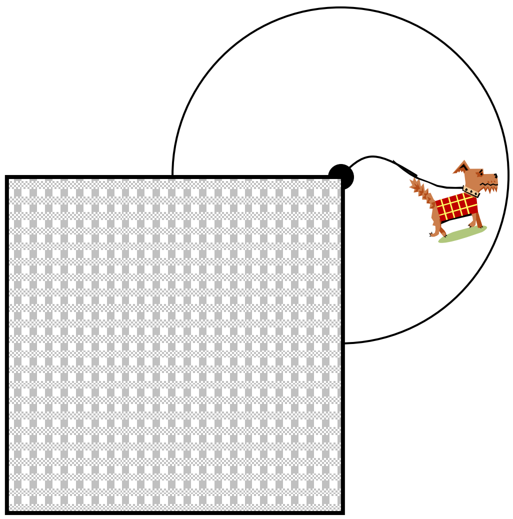
- Approximate the maximum height of the performer.
  14.5 meters

- About how long does it take for the performer to reach his/her maximum height?
  3 seconds

- Approximately how long is the performer in the air before landing?
  Almost 7 seconds

Woof! Woof! Name: ______________________
Launch Date: ________________________

Katie's house is 50 ft by 50 ft. Her dog, Broody, is attached to a rope at the corner of the house. His rope reaches halfway down the house. How much area does Broody have in which to play?
Use mathematics to explain your answer. Use words, symbols or both in your answer.



Woof! Woof! Name: ______________________
Launch Date: ________________________

Katie's house is 50 ft by 50 ft. Her dog, Broody, is attached to a rope at the corner of the house. His rope reaches halfway down the house. How much area does Broody have in which to play?
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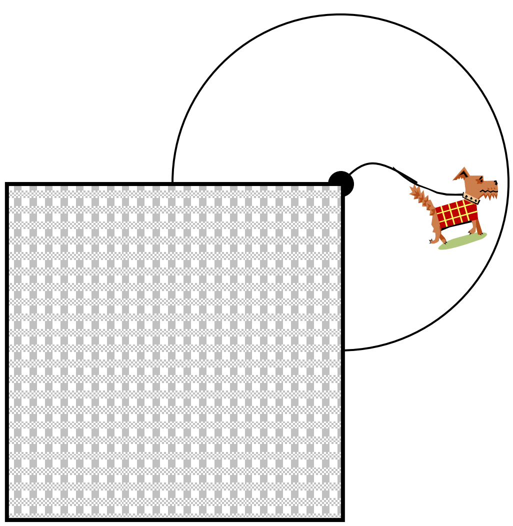


Woof! Woof!
Launch

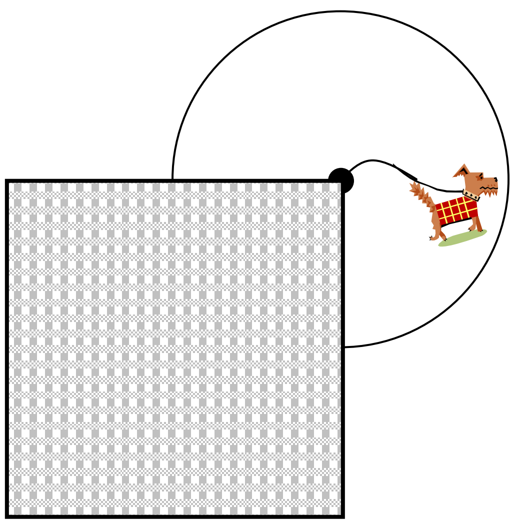
Name: ___ANSWER KEY________
Date: __________________________

Katie's house is 50 ft by 50 ft. Her dog, Broody, is attached to a rope at the corner of the house. His rope reaches halfway down the house. How much area does Broody have in which to play?
Use mathematics to explain your answer. Use words, symbols or both in your answer.

$$A_{\text{Circle}} = \pi r^2$$

$$A_{\text{Dog Run}} = \frac{3}{4} A_{\text{Circle}}$$

$$= \frac{3}{4}\pi\left(25^2\right) \approx 1472.6216 \text{ ft}^2$$



Cow Pasture
EI-EI-O

Name: ______________________
Date: _______________________

Old McDonald was so inspired by Mrs. McDonald's new fencing on her garden that he decided to make a new fence for his cow, Bessie. He had $3,000 and was able to buy 80 yards of fencing. He wants to make sure to maximize the area in which Bessie has to roam and eat grass.

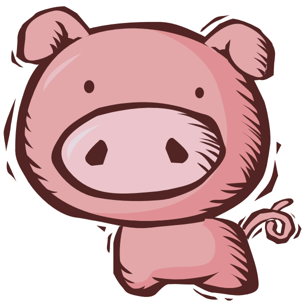
- How much does the fencing cost per yard? ___________

- Help Old McDonald find the dimensions (length and width) to create Bessie's pen.
  - L: _____ W: ______
  - Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

- What is the area enclosed? _________________

Pig Pen
EI-EI-O

Name: ______________________
Date: ____________



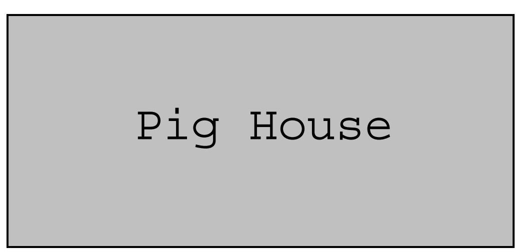
Not to be unfair to his pigs, Old McDonald decided to redo the fence of his pig pen. The pig house, where the pigs sleep, is 30 foot wide by 10 foot deep. To save fencing, Old McDonald wants to use *part of* or *all of* the 30 foot wall as one side of the pen.

He bought 72 feet of fencing for $432. He needs to know what dimensions will produce a maximum area in which the pigs can roam.

- How much does the fencing cost per foot? ___________

- What are the dimensions (length and width) which Old McDonald should use?
    - L: _____ W: ______
    - Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

- What is the area enclosed? ______________



Billy the Goat
EI-EI-O

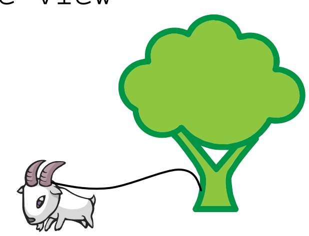
Name: ______________________
Date: _______________________

Old McDonald has a goat, Billy, whom he likes to take out of the pasture and tie him to the big oak tree in the front yard. Billy can then watch the cars go by on the road. Every time a car drives by, Billy runs around the tree. Each time the rope wraps around the tree, it is shortened and Billy has a smaller circle in which he can run. Knowing that the rope is 45 feet long and the oak tree has a diameter of 3 feet, answer the following questions. Use $\pi = 3.14$.

1. If Billy sees one car, what is the new length of his rope? _________
2. After three cars pass by, what is the length of Billy's rope? _______
3. How many cars can pass by before Billy has no rope left? ________
4. Complete the following table:

# Cars	Length of rope (ft)	Area of circle (sq ft)
0		
1		
2		
3		
4		

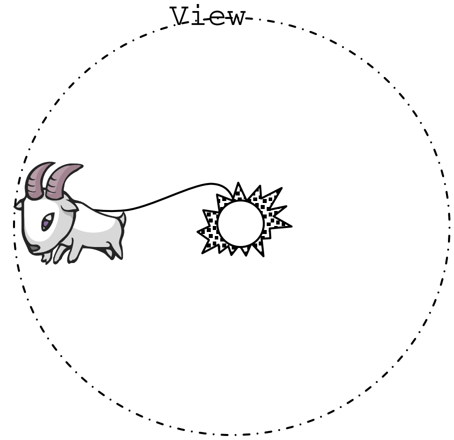
Side View



5. Write an equation relating the number of cars to the length of the rope.

6. What type of relationship exists between the number of cars and the area of the circle

Top View



Cow Pasture

Cost Per Yd: $37.50

Dimensions: 20 x 20 yds

Max Area: 400 sq yds

Pig Pen

Cost Per Yd: $6.00

Dimensions: 24 x 24 ft

Max Area: 576 sq ft

Billy the Goat

1. 35.58 ft
2. 16.74 ft
3. Between the 4th and 5th car.

4.

# Cars	Length of rope (ft)	Area of circle (sq ft)
0	45	6358.5
1	35.58	3795
2	26.16	2148.8
3	16.74	879.91
4	7.32	168.25

5. $y = 45 - 9.42x$
6. A quadratic relationship.

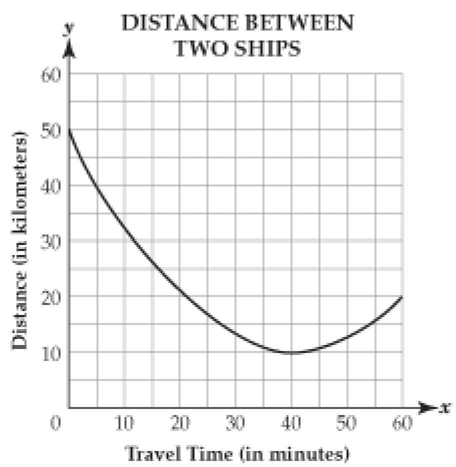
For this problem, students may want to include the radius of the oak tree when they calculate the area of the circle. They may also subtract out the area of the tree when calculating the area in which the goat can run. We have not done that in order to keep the problem on the Algebra I level.

Sample Formal Assessment Problems

1. Elizabeth joins a CD buyer's club. She receives 10 free CDs when she joins this club. She must buy 3 CDs each month.

- Write an equation that represents the number of CDs (*y*) Elizabeth will receive from the CD buyer's club after *x* months.
- What is the *y*-intercept of your equation? What does the *y*-intercept mean in the context of this problem?
- Elizabeth wants to receive no more than 55 CDs from this club. What is the maximum number of months Elizabeth will remain in the CD buyer's club? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

2. Two ships travel in the Chesapeake Bay. The graph below shows the distance between the two ships during the time the ships travel.



How many minutes have the ships traveled when the distance between them is the shortest?

a. 10 minutes
b. 20 minutes
c. 40 minutes
d. 60 minutes

3. Toy blocks are used to build a tower. The surface area and volume of the tower built with these blocks is shown in the table below.

MODEL TOWER VALUES

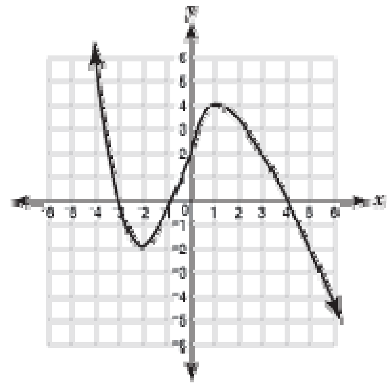
Number of Blocks	Surface Area (in square centimeters)	Volume (in cubic centimeters)
1	18	4
2	30	8
3	42	12
4	54	16
5	?	?
6	?	?

- Complete the table to determine the surface area and the volume for 5 and 6 blocks.

- Write an algebraic expression to represent the relationship between the number of blocks and the surface area of the tower. Use mathematics to justify your answer.

- If 10 blocks are used, what is the surface area and the volume of the tower? Use mathematics to explain how you determined your answers. Use words, symbols, or both in your explanation.

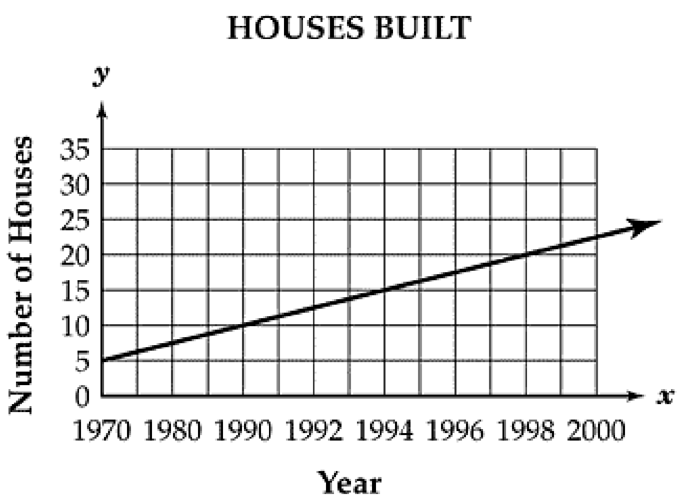
4. Look at the function that is graphed below.



Which of these represents the number of zeros of this function?

a. 0 b. 1 c. 2 d. 3

5. The graph below shows the number of new houses built in a town from 1970 to 2000.



The mayor of the town used the graph to claim that between 1970 and 2000 the number of new houses built increased at a constant rate. Is the claim valid?

A. It is valid because the graph shows a constant rate of change.

B. It is valid because 30 years is long enough to evaluate the increase.

C. It is not valid because the scale on the vertical axis is inappropriate.

D. It is not valid because the scale on the horizontal axis is inappropriate.

6. The table below shows a relationship between x and y.

x	y
2	5
3	7
4	9
5	11

Which of these equations represents this relationship?

A. $y = x+3$

B. $y = x+4$

C. $y = 2x+1$

D. $y = 2x-1$

7. The table below shows a relationship between x and y.

x	1	2	3	4
y	2	5	10	17

What is the value of y when x is 10?

8 **Look at the pattern below.**

5, 10, 20, 40, . . .

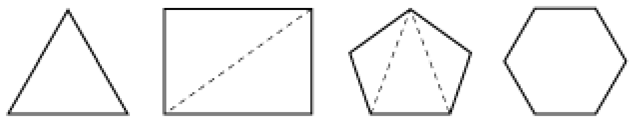
If this pattern continues to increase, what is the 6th term?

A 60

B 80

C 160

9. A triangle, a quadrilateral, a pentagon, and a hexagon are shown below. By drawing a diagonal from 1 vertex, the quadrilateral is divided into 2 non-overlapping triangles. Since the sum of the angle measures of a triangle is 180°, the sum of the measures of the quadrilateral is 360°. By drawing the diagonals from 1 vertex, the pentagon is divided into 3 non-overlapping triangles.



Complete the following:

Draw the diagonals from 1 vertex of the hexagon so that the hexagon is divided into non-overlapping triangles.

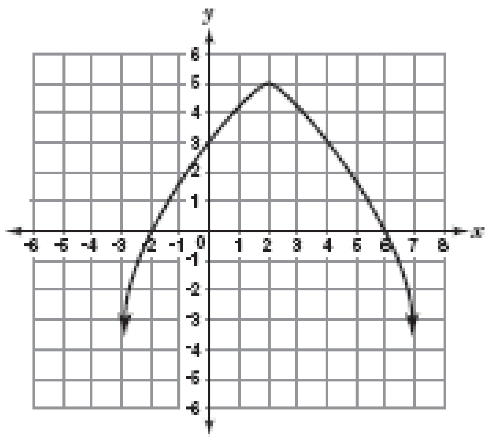
Use the polygons above to complete the table.

Polygon	Number of Sides	Number of Non-Overlapping Triangles	Sum of Angle Measures
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5	3	
Hexagon			

Describe how the number of sides of each polygon is related to the number of non-overlapping triangles. Use mathematics to justify your answer.

Describe how the number of non-overlapping triangles in a polygon is related to the sum of its angle measures. Use mathematics to justify your answer.

10 Look at the function below.



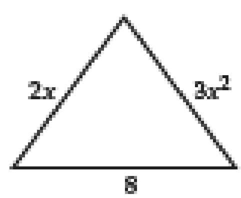
What is the maximum value of this function?

A 2

B 5

C 6

11 Look at the triangle below.



Which expression represents the perimeter of this triangle?

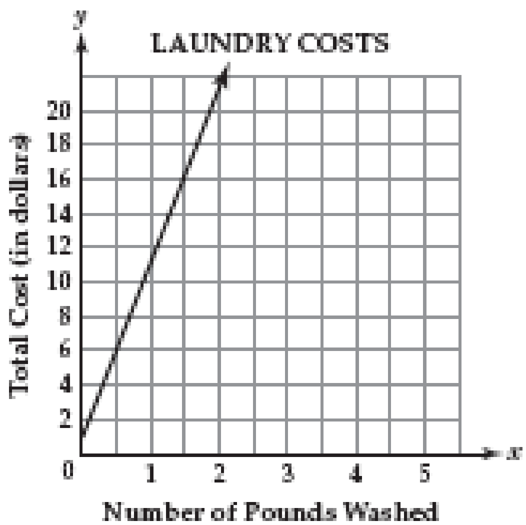
A $3x^2 + 2x + 8$

B $3x^2 - 2x - 8$

C $(3x^2)(2x)(8)$

12 A laundry service charges Wes $10.00 plus an additional $1.25 per pound to wash his laundry.

Which graph represents the relationship between the number of pounds of laundry washed (x) and the total cost (y)?



C